


## PERCEPTIONS OF PRE-SERVICE TEACHERS REGARDING THE MODEL ELICITING ACTIVITIES

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*This study aims to examine the perceptions of pre-service mathematics teachers about mathematical modelling activities. Participants of the study comprised 23 pre-service mathematics teachers who undertook a course on mathematical modelling. A 12-hour mathematical modelling course revealed the perceptions of participants. After it, the participants found/developed a modelling problem and explained why they evaluated it as a model eliciting activity (MEA). MEA found/developed by participants were examined by taking the principles of developing MEA into consideration. Results showed that many participants considered having more than one solution, the principle-based on real-life context, and suitability for group work for MEA. The participants did not focus on the model documentation principle. Based on this result, faculty members can help pre-service teachers by using activities that can be considered appropriate for this principle in modelling education.*

**KEYWORDS:** Mathematical Modelling, Modelling Principles, Model Eliciting Activities, Pre service Teachers

### INTRODUCTION

In the most general sense, mathematical modelling is the process of mathematically expressing problem situations encountered in real life. If this definition is further expanded, it is a process that consists of six stages in which real-life problems are mathematized and solved, and the solution is evaluated ([Haines & Crouch, 2007](#)). These six stages are: expressing the real-life problem, formulating a model, solving this model mathematically, interpreting and evaluating the solution, and correcting the model if necessary. Since some researchers

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emphasize that a report should be prepared at the end of this process, the reporting stage can be added as the last stage. [Blum \(2011\)](#) explains these stages as follows: The problem solver creates a situation model to understand the real-life problem. In the second stage, the problem situation is structured further, and a real model of the situation is created. The mathematization stage is the transformation of the real model into a mathematical model that includes mathematical objects such as variables and equations. Mathematical results are obtained by carrying out mathematical work on this model. These mathematical results are employed in the next step to interpret the real model and the results are verified. After this stage, the problem solver enters this cycle again if necessary, and the process is repeated, possibly until a satisfactory result is obtained. In the reporting phase, an explanation of the model in the form of a report, letter, etc. is presented to the relevant people regarding the solution of the problem. This follows from the realization of the real-life principle inherent in mathematical modelling activities.

When mathematical modelling definitions and processes are examined, it can be noticed that it shows some similarities with ‘problem’ and ‘problem solving’. However, model eliciting activities (MEA) have slightly different characteristics from the problem definitions given in the literature. According to [Lesh, Hoover, Hole, Kelly, and Post \(2000\)](#) and [Lesh, Cramer, Doerr, Post, and Zawojewski \(2003\)](#), MEA should comply with six principles: meaningfulness to the individual or principle of reality, model construction principle, self-assessment principle, model documentation principle, model generalization principle, and effective prototype principle. These principles are explained as follows:

- **Meaningfulness to the individual or the principle of reality:** The basic criterion for this principle is the possibility of encountering the situation found in the problem in real life. Students will interpret the given situation within the problem based on their past and personal experiences.
- **Model construction principle:** The problem situation should lead the students to feel the need to formulate a model or to modify, expand, or correct an existing model.
- **Self-assessment principle:** Students should be able to decide for themselves how effective the model they formulated with the modelling activity is.
- **Model documentation principle:** “Does it give information about how the students reach the answer or the solution they have produced, or the thinking system of the students?” are fundamental questions whose answers are sought in this principle.
- **Effective prototype principle:** The model created by the students should

be as simple as possible and should be like a prototype that can be adapted to similar problem situations. The model should hold characteristics that enable it to be remembered and used by the student when a similar situation is encountered.

- Model generalization principle: In connection with the previous stage, the model should be generalizable or adaptable to similar problem situations.

These six principles which were put forth after multitiered teaching experiments, serve to demonstrate the conceptual structures developed by students, which is the most important criterion in revealing the effectiveness of an MEA (Lesh et al., 2000).

## REVIEW OF LITERATURE

MEA are useful for both teaching and evaluating the development of students and teachers, as well as for assessment and research purposes. Thus, the aims of the studies in the context of mathematical modelling have spread to a wide range. In a study conducted with the model eliciting principles, Dede, Hıdıroğlu, and Güzel (2017) found that in the modelling activities created by pre-service teachers working in groups, reality and model eliciting principles play a binding role and the principles of self-assessment and model documentation are directly related to each other. In addition, model generalization and effective prototype principles are principles associated with others, and the researchers conclude that they can be effectively produced by observing future practices. Apart from these, when studies on modelling (Chamberlin & Moon, 2005; Lesh & Caylor, 2007; Lesh et al., 2000) are examined, it is seen that modelling activities are expected to be realistic and open-ended problems. In addition, it is emphasized that the problem situation should have characteristics such as allowing different solutions, requiring higher order thinking skills, enabling students to learn and self-assess, allowing group work, and revealing the existence of relationships between different disciplines.

As the uses of mathematical modelling in the field of education are examined, it is noteworthy that various perspectives or approaches are used. To make a general classification for these approaches, it can be said that mathematical modelling is used for instruction and research purposes (Kaiser, Sriraman, Blomhøj, & Garcia, 2007). The first approach defines the teaching approaches related to modelling and is significantly influenced by the theoretical structures in its background. The second approach guides the development of modelling skills and studies in this field. Many researchers (Galbraith, 2012; Julie & Mudaly, 2007) consider modelling as a tool and a subject. While real-life problems are used in modelling-as-a-tool for students to

learn mathematical subjects and increase their motivation; the purpose of modelling-as-a-subject is to increase students' ability to solve problems that they will encounter in real life, that is, their mathematical modelling skills. In another study, [Kuntze \(2011\)](#) examined modelling problems in two groups: Tasks with high modelling requirements (with at least one transformation between the given situation and the mathematical model, and allowing different solutions) and tasks with low modelling requirements (where the mathematical model is given before and involves processes where the transfer between the real-life situation and the mathematical model is less important, and only one correct answer is possible). However, it would not be very accurate to separate these classifications with strict lines. For example, considering the classifications of modelling for instruction and research purposes and modelling as a tool or a subject, modelling-as-a-tool can be evaluated within instructional modelling, and modelling-as-a-subject can be evaluated within research-oriented modelling. Tasks with high modelling requirements and tasks with low modelling requirements can be included in any of these categories.

In recent years, mathematical modelling studies have had a crucial role in mathematics education. This importance manifests itself with the emphasis on mathematical modelling in the curricula of countries and in the studies conducted in mathematical modelling. For instance, in the new mathematics curriculum announced by the (Turkish) Ministry of National Education [MEB \(2018\)](#), it is stated that individuals who can use mathematics in modelling and problem solving are needed more than ever and learning outcomes for modelling are included in the curriculum. Similarly, in the Common Core State Standards used in America, being mathematically proficient is explained as follows: students can solve the problems they encounter in daily life by using their mathematical knowledge according to their grade levels ([Common Core State Standards Initiative, 2021](#)). When it is taken into consideration that mathematical modelling is defined as the process of converting real-life problems into mathematical language using mathematical terms ([Cheng, 2001](#)), emphasis on mathematical modelling emerges in these standards as well.

The introduction of mathematical modelling into mathematics curricula has also affected undergraduate teacher training programmes and mathematical modelling has taken its place among the compulsory field courses in the new mathematics teaching undergraduate programmes announced by the (Turkish) Council of Higher Education. This has revealed several competencies such as the ability of mathematics teachers to carry out mathematical modelling or use it as a teaching activity. Up to this point, various studies have been carried out, which include teachers and pre-service teachers as participants, and various results have been obtained on their views and applications of mathematical modelling. For instance, [Ozer and Guzel \(2016\)](#)

found that despite knowing the differences between modelling problems and other problem types, pre-service teachers who were trained in mathematical modelling had limited perception in this context. Similarly, in the study of Yanık, Bağdat, and Koparan (2017) pre-service teachers who received mathematical modelling training stated that mathematical modelling problems differ from the problems in the textbooks in many aspects. Cakmak-Gurel and Isik (2018), in their study examining the mathematical modelling competencies of pre-service teachers who participated in the mathematical modelling learning environment and of those who did not, found that there were significant differences in favour of pre-service teachers who participated in the learning environment in terms of simplification/structuring, mathematization, and interpretation competencies. This situation was interpreted as the organized learning environment having a positive effect on the mathematical modelling competencies of pre-service teachers. In connection with this result, pre-service teachers think that their lack of extensive experience with mathematical modelling in their own mathematical preparation constitutes an obstacle to their ability to organize pedagogical activities, even though they see modelling as an important skill to be developed in mathematics teaching (Manouchehri, Yao, & Saglam, 2018). Therefore, it is important to include modelling activities in studies to be carried out on mathematical modelling (Holmquist & Lingefjärd, 2003).

As teachers carry out activities in which they can formulate their own models, they will become aware of their own thinking processes, be able to test and verify these processes in the modelling cycle, and thus be able to develop knowledge of learning and teaching for education in accordance with the requirements of the current century (Doerr & Lesh, 2011). In another study, Kuntze (2011) took the opinions of mathematics teachers and pre-service mathematics teachers about classroom tasks that require (high and low-level) modelling. Results showed that pre-service teachers preferred activities that require low-level modelling to activities that require high-level modelling. The researcher thinks the confusion created by mathematical precision in mathematical modelling activities may be one of the reasons for this result. Teachers, on the other hand, approach more positively to problem situations that require high-level modelling. This is explained by the teachers' awareness of the learning opportunities created by the high-level modelling activities.

The aim of this study is to examine the perceptions of senior pre-service mathematics teachers towards mathematical modelling activities after 3 weeks of mathematical modelling training by considering the modelling activities they have developed/found themselves. Studies on pre-service teachers' perceptions of mathematical modelling activities were mostly based on the views of pre-service teachers, but the compatibility of their views and practices was

not taken into serious consideration. However, how the pre-service teachers' views are being put to practice is as important as their views. While many teachers recognize the value of their students' participation in mathematical modelling, few have opportunities to get experience in modelling and many teachers are unsure of how to teach it (Hernández, Levy, Felton-Koestler, & Zbiek, 2017). It is expected that this study will contribute to the literature since it includes views of pre-service teachers as well as the modelling activities used by them to verify their views. The comparative importance of criteria from the perspective of pre-service teachers when selecting a modelling activity also provides guidance to teaching staff working in education in this field.

## RESEARCH QUESTIONS

For the purposes of this study, answers to the following questions are sought:

1. Considering the principles for developing MEA, how adequate is the MEA chosen/developed by the participants after their mathematical modelling training?
2. How consistent is the reasoning of the participants for the adequacy of their MEA (chosen/developed by the participants after their mathematical modelling training) considering the principles for developing MEA?

## RESEARCH METHODOLOGY

An explanatory case study method was used in this study and is descriptive in nature. It is used to give information about the case/s that are the subject of the study. The main reason for structuring the research design in this way is that the participants are pre-service teachers studying in a mathematics teacher training programme and the modelling activities they have developed/found are analysed according to the principles accepted in the body of literature. Therefore, a case is analysed, and a theoretical background is used for the analysis.

### The Study Group

The participants of the study were 23 senior pre-service mathematics teachers, aged 22-23 years out of which, 19 were females and 4 were male. The students were studying in a large-scale public university at the Department of Mathematics Education in Ankara during the spring semester of the academic year 2018-2019. Participants had completed all other courses which included mathematics field and field education courses (Analysis I-II-III-IV, Linear Algebra I-II, Differential Equations I-II, Analytical Geometry I-II, Abstract Mathematics I-II, Introduction to Education, Instructional Principles and

Methods, Mathematics Teaching and Learning Approaches, etc.) in their program except for the final semester courses. The convenience sampling was used in determining the participants. The main reasons for selecting the participants of the study were that a) the pre-service teachers were not attending the new curriculum, in which 'mathematical modelling' was not included within the scope of compulsory courses for pre-service mathematics teachers b) that they had not taken any courses for mathematical modelling before c) that they were in the last year of their education and d) that they completed most of the courses for field education. Thus, the source of their perception on mathematical modelling had mostly stemmed from the training included in this study.

### **The Context of The Study and Data Collection Tools**

To reveal how pre-service teachers perceive mathematical modelling problems, a 12-hour mathematical modelling training was carried out in a lesson given by the researcher. The reason for this training is that the students had not received any modelling training before. At the beginning of this training, the problem and problem types were introduced, and non-routine and real-life problems were emphasized. After that, the examples of MEA were presented and the differences and common aspects of these problems with other problem types were discussed. Then, three mathematical modelling activities were carried out. For this, students worked in groups to prepare reports on the solution approaches they suggested to problems and presented these reports in the class. Finally, the features of these activities were emphasized and their advantages and disadvantages in terms of mathematics teaching were discussed. The criteria that should exist in a mathematical modelling activity were discussed over these activities, but no criterion was specifically stressed.

The data of the study consists of the answers to a mid-term exam question asked to the pre-service teachers. Before the mid-term exam was conducted within the course, students were asked to find/develop a modelling problem. During the exam, they were asked to indicate why they selected/developed this problem, and why they believed the problem they chose/developed was a modelling activity. The basic reason to collect pre-service teachers' answers in an exam environment is to make them think more selectively/carefully in finding/developing these activities. Thus, it was thought that what attracted their attention more during the training would be revealed and that they would choose activities that fit their perception.

### **Analysis of the Data**

The obtained data were analysed through document analysis. During the analysis, MEA principles previously defined by [Lesh et al. \(2003\)](#) and the basic criteria that should apply to a modelling activity stated in the literature

(being realistic and open-ended problems, the problem situation allowing different solutions, requiring higher-order thinking skills, allowing students to learn and self-assess, allowing group work, and revealing the existence of relationships between different disciplines) were considered. Therefore, the data obtained were subjected to descriptive analysis.

The modelling activities developed by the students were evaluated using the rubric that [Urhan and Dost \(2018\)](#) used to evaluate the modelling activities that were in high school textbooks, consisting of ‘completely appropriate’, ‘partially appropriate’, ‘inappropriate’ and ‘indeterminable’ dimensions. Every modelling activity developed/found by the pre-service teachers was included in the data analysis. Four randomly selected modelling activities developed/found by pre-service teachers were evaluated by another expert in the field in terms of the consistency between model eliciting principles and the justifications presented by pre-service teachers as to these models having the properties that should be found in a modelling activity and 0.93 (Cohen’s Kappa) consistency was found.

FINDINGS OF THE STUDY

In this section, the problems chosen as modelling activities by the pre-service teachers will be evaluated in terms of MEA principles. Table 1 shows the number of MEA which were developed or found from the literature by the pre-service teachers and their classification according to [Kuntze \(2011\)](#) as to whether these activities require low or high level modelling. Four of the modelling activities found in the literature were written by nine participants. One of the activities was written by three pre-service teachers and the others by two pre-service teachers each.

**Table 1**  
**Sources of Modelling Activities Found/Developed by Pre-service Teachers and the Level Required for the Modelling.**

	Total Number of Activities	Activities Requiring High-Level of Modelling	Activities Requiring Low-Level of Modelling
Activities developed by pre-service teachers	6	4	2

*Continued on next page*



*Table 1 continued*

Activities found from the literature	17	6	6
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Table 2 and Table 3 consist of the evaluation of the MEA that pre-service teachers found/developed according to the principles of model developing. When Table 2 is examined, it is seen that all activities are fully compatible with the reality principle. However, regarding the model documentation principle, (different) activities have evaluated at all three dimensions. The adequacy of participants' models according to the effective prototype principle was evaluated under 'indeterminable'. The models to be created for the activities both developed by the participants and found from the literature were not implemented. Hence, it was decided that how prototypical or memorable they would be could not be determined, as in the study of [Urhan and Dost \(2018\)](#). Differing from Table 2, Table 3 shows activities that are partially suitable and not suitable for the principle of reality. Further, the number of activities that are not appropriate for the model documentation principle is quite high.

**Table 2**

**Evaluation of Modelling Activities Developed by Pre-service Teachers According to Principles for Developing MEA.**

Principles for Developing MEA	Fully Compatible (f)	Partially Compatible (f)	Incompatible (f)	Indeterminable (f)
Reality	6	-	-	-
Model Construction	4	2	-	-
Self-Assessment	4	2	-	-
Model Documentation	2	2	2	-
Model Generalization	4	1	1	-
Effective Prototype Principle	-	-	-	6

**Table 3**  
**Evaluation of Modelling Activities Found in the Literature by Pre-service Teachers According to Principles for Developing MEA .**

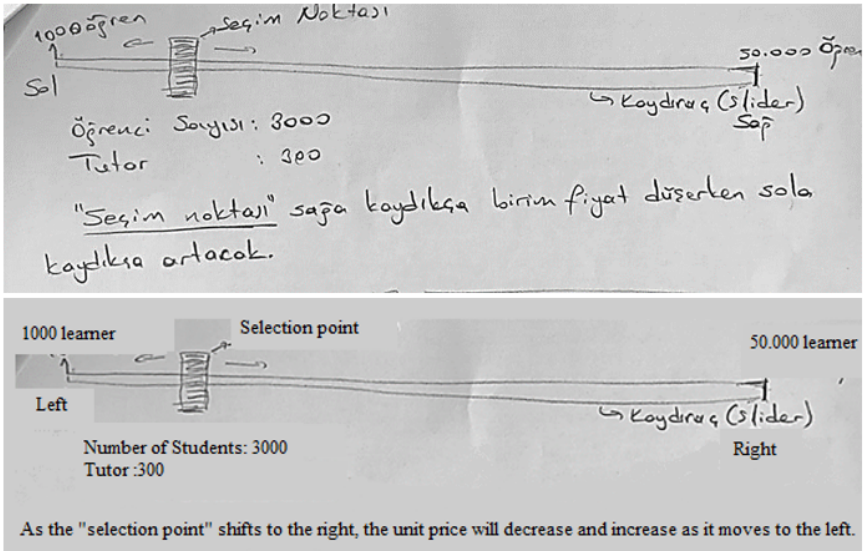
Principles for Developing MEA	Fully Compatible (f)	Partially Compatible (f)	Incompatible (f)	Indeterminable (f)
Reality	5	4	3	-
Model Con- struction	8	3	1	-
Self- Assessment	10	1	1	-
Model Doc- umentation	4	2	6	-
Model Gen- eralization	5	6	1	-
Effective Prototype Principle	-	-	-	12

**Pre-Service Teachers’ Examples of MEA and their Evaluation Processes**

In this section, the MEA examples given by the participants were examined. First, MEA examples developed by the participants were given. Second, activity examples found from the literature were presented. For each example, first the text of the activity; then the examination as to whether it is an MEA; and finally, the justifications of the participant as to why the example is an MEA is presented. MEA developed by one of the participants is as follows:

*A firm has developed an online exam system. They want to charge per use rather than selling the system. For example, 300 TL for a 3,000-student package, 500 TL for a 5,000-student package. However, they want to give a more affordable price for students who buy a higher package, like charging 450 TL for a package of 5,000 students while a package for 3,000 students is 300 TL.*

This example is fully compatible with the real-life principle. It is known that companies have such sales policies. It is an example that is fully compatible with the model developing principle as well. The slider presented by the student in Figure 1 is a visual expression of a mathematical model. Although there is information about the number of sales, as many variables



**Figure 1. Image of the MEA example created by the pre-service teacher (original in Turkish and the translated version)**

will be based on student assumptions, it will be difficult for the student to ascertain the effectiveness of the model on their own. The question is not very clear. Therefore, the example fits the self-assessment principle partially. The model is also partially compatible with the model documentation principle. It is said that the system will be charged per use. However, no document (such as preparing a report for the company executives as the conclusion of the consultancy service) is requested. The model is fully compatible with the model generalization principle because the model can be used by companies producing licensed software to determine license fees depending on the number of users.

The pre-service teacher explained why this problem was suitable for MEA in the following way:

*"It is a part of daily life, it is multi-disciplinary, it can be expressed mathematically and has more than one solution."*

Another modelling activity developed by the pre-service teachers is as follows:

*The government wants to build a third bridge for Istanbul. To do this, they want to make a contract with one of the 3 foreign companies that work with the construct-operate-transfer model. The first company wants a deal of 29-year toll and a guarantee*

*of 35,000 vehicles per day. The second company wants a deal of 39-year toll and a guarantee of 30,000 vehicles per day. The third company wants a deal of 49-year toll and a guarantee of 25,000 vehicles per day. If the requested vehicle crossing is not provided, the state will compensate the companies for the difference. For this, the state requests that the companies allow free pass on special days. In turn, companies request 5%, 10%, and 15% increase in tolls, respectively. Given these facts, produce a solution method that shows which company should get the contract. Also, decide whether it is appropriate to give money paid by the country out to foreign companies.*

The problem takes its context from real life. The solution of the problem allows developing model. Since the problem is clear enough, the model is compatible with the self-assessment principle to the extent that it allows the comparison of the companies' offers. The model complies with the model documentation principle, with the statement "Given these facts, produce a solution method that shows which company should get the contract". The requested model can be used in different situations (private power plants, highways, tunnels) as well. Therefore, the model fits the model generalization principle. The pre-service teacher explained why this problem is an MEA as follows:

*This question concerns daily life problems since it is about the construction of a bridge and making a tender. There are many solution methods because we have variables such as the year, toll fee, and a raise at the end of the year. Thus, there will be multiple results. When this question is given to a student, they will be led to think mathematically without asking for any support to find a solution. The result of this question will be remembered repeatedly. It is a question in which students can make use of methods such as equations, formulas, and tables to reach a solution.*

Although the participant thinks that the result would be remembered repeatedly, this claim is hypothetical as he did not implement the problem. Apart from this, the last sentence in the participant's modelling activity aims to question the acceptability of the model by the public. Thus, it emphasizes the socio-critical aspect of modelling as previously indicated in the literature. In this genre, which emerged as one of the subgroups of modelling, the acceptability of the model by the public is also taken into consideration, besides being critical about the model, its validity, or its assumptions (Kaiser et al., 2007).

One of the examples chosen by the participants from the literature is as follows: "The biggest bridge in the world is the Hongzhou Bridge, located in the east of China and 36 km long. Assuming that there is a traffic jam along the whole bridge, how many cars could there be on the bridge?" (Peter-Koop, 2004). This problem takes its context from real life as well, because in cities with high population density it is possible to see kilometres long traffic jams under heavy traffic conditions. Since the solution of the problem allows model formulation, it fits the model construction principle as well. The effective-

ness of the model can be determined by the students; therefore, it fits the self-assessment principle. The model does not follow the model documentation principle because what is requested is an answer, not an explanation for the reasoning. The model is partially compatible with the model generalization principle. The model can be used in very similar situations. The pre-service teacher explains why this problem is an MEA as follows:

*It is an open-ended question. The answer cannot be immediately obtained. It is a problem we might encounter in daily life. The solution is not clear. There is no single answer, and students can work in groups while thinking about the answer to the question. The fact that the answer may vary according to different car lengths (making inferences based on assumptions) allows the problem to have more than one solution. It requires the use of different measurement units and formulas such as arithmetic mean and summation.*

An MEA found from the literature by another participant is as follows:

*A woman feeds daily on four basic food items: muffins, chocolates, soda, and cakes. The cost of a muffin is 0.5 TL, a bar of chocolate costs 0.3 TL, a bottle of soda costs 0.5 TL, and a slice of cake costs 0.8 TL. This woman should consume 600 calories, 6 grams of chocolate, 8 grams of sugar, and 5 grams of fat daily. Nutritional values are given in the Table 4. Consulting the table below (Table 4), how much of each item should this woman consume each day to ensure minimum spending? (Winston, 2003).*

**Table 4**

**Table Created by Pre-service Teacher for Modelling Activity.**

Food	Calories	Chocolate (gr)	Sugar (gr)	Fat (gr)
Cake	500	3	2	2
Chocolate	600	2	2	4
Coke	150	0	4	1
Pastry/Slice	450	0	4	5

The problem is appropriate for MEA because it is suitable for different solutions; suitable for mathematization; has properties that can allow group work; leads the solvers into a cyclic process. However, the real-life context appears to be weak. In real life, it would be irrational to feed on these four food items, and the values given do not match reality. Although these elements can be easily changed, the participant may not have noticed the necessity of this due to their habits from previously encountered problem types in textbooks. The pre-service teacher gave this explanation for this problem he found in the literature: "It can be associated with real life, different solutions can be

produced for it, it is suitable for solution with group work, it demands mathematical expressions to be formulated to reach the solution, the solution would be produced in a cycle, the student is asked to link together their pieces of knowledge.”

The justifications given by the pre-service teachers in their explanations for why the modelling problems they wrote are modelling activities and the consistency of these with the modelling activities are given in Table 5. In some cases, the justifications of the participants do not reflect all the features of the MEA they have developed/found. In some other cases, their justifications are not part of the MEA. While creating Table 5, problems that were repeated were also taken into consideration, because the justifications presented by different students were different.

**Table 5**  
**The Justifications for the Problems Developed/Found by the Participants for Being a Modelling Activity and their Consistency with the Structure of Modelling Activities.**

Rationale	Frequency (f)	Consistency (f)
Multiple Solution	21	19
Real-Life Context	20	15
Suitability For Group Work	12	11
Mathematization	10	10
Suitability For Self-Assessment	9	9
Appropriateness For Model Construction	8	7
Requiring Assumptions	5	5
Memorability	5	-
Being An Open-Ended Problem	4	3
Requiring High-Level Thinking Skills	3	2
Interdisciplinary Knowledge	2	2
Applicability To Different Situations	2	2
Requiring A Cyclic Process	1	1
Generalizability	1	-

As Table 5 is examined, “Multiple solutions”, “Real-life context”, “Suitability for group work”, “Mathematization”, “Suitability to self-assessment” and “Appropriateness for model construction” are the most frequent justifications for the example of being an MEA. On the other hand, “Interdisciplinary knowledge” and “Applicability to different situations”, “Requiring a cyclic

process" and "Generalizability" emerged as the least frequent justifications. Participants' justifications are often consistent with the MEA they have developed/found. The discrepancy between justification and consistency is most evident in the "real-life context". Here, examples that do not completely fit real-life context were considered inconsistent. Problem types inconsistent with real-life context were exemplified in the activity about the nutrition program given earlier. Since the consistency of the "Memorable" justification cannot be determined for the examples given by the participants, the consistency line is left blank for this item.

## DISCUSSION AND CONCLUSION

The instructional benefits of modelling and the problem-solving repertoire it provides to individuals in daily life have led to an increase in the importance given to this subject in mathematics education. Accordingly, the increase of studies in this field has also produced a wide literature for mathematical modelling. One of the pieces of knowledge that emerged in the literature of mathematical modelling is what properties a modelling activity should have. This study was carried out to examine the perceptions of pre-service mathematics teachers towards mathematical modelling activities. Through document analysis, it was examined which of the said properties were taken into consideration by pre-service teachers in the modelling activities they developed or found from the literature.

Considering the modelling problems prepared by the pre-service teachers six pre-service teachers chose to develop their own modelling problems. Since developing a modelling problem is a very difficult process, even the attempt makes us think that the participants feel competent in this regard. The mathematical modelling practices during the modelling training may have contributed to this result. As previously mentioned, it is very important to include modelling activities in studies related to modelling (Doerr & Lesh, 2011; Holmquist & Lingefjärd, 2003).

Considering the level of the modelling activities that were prepared, it is observed that there were more problems requiring high-level modelling skills. When studies conducted in the literature are examined (Kuntze, 2011), it is seen that pre-service teachers prefer activities that require low-level modelling. However, these choices of the pre-service teachers might be based on the facts i) they prepared these activities as an answer to an exam question, ii) that they paid attention to what properties MEA should have, and iii) they did not have to apply them in a classroom environment.

When the modelling activities developed/found by the pre-service teachers are examined according to the principles for developing MEA, the real-life con-

text emerges as the criterion that pre-service teachers' pay the most attention to, especially in MEA they developed themselves. It is noteworthy that this criterion was prominent in previous studies conducted with pre-service teachers (Dede et al., 2017; Ozer & Guzel, 2016). However, for MEA that were chosen from the literature, this criterion was not given the same importance. This might be caused by pre-service teachers trusting the source of the modelling activities they encounter in their online searches, or by pre-service teachers not inquiring thoroughly. One of the criteria that most MEA in the literature are completely appropriate for is "self-assessment". The reason for this might be that the MEA is solved by the pre-service teachers, that the MEA is evaluated for its suitability for the students, and that the problem is sufficiently clear. On the other hand, the model documentation principle emerges as a criterion not given enough importance, in both MEA developed by the participants and MEA chosen from the literature. The model documentation principle is an element in mathematical modelling that requires the explanation of the solution process and the thinking processes behind the solution. In fact, with these properties, it is an element that distinguishes MEA from other non-routine problems. Thus, it can be considered as a new feature to tackle for pre-service teachers. Consequently, it is an expected result that it was not given much consideration.

When Table 5 is considered, it is seen that "allowing more than one solution method" and "real-life context" are the most frequent justifications. Real-life context has been previously examined in the principles of MEA. Allowing more than one solution is also a feature that should be in a modelling activity. Since this is a feature found in non-routine problems, the participants were already familiar with it and it is also highlighted for MEA in the literature (Mousoulides, Sriraman, & Christou, 2007). Similarly, suitability for group work, mathematization, and suitability for self-assessment are some of the more frequent answers. "Requiring assumptions" is not a justification highly emphasized by pre-service teachers, even though it is related to allowing more than one solution. Since especially the modelling activities that the pre-service teachers chose from the literature generally contain all the data, having more than one solution may have been considered as reaching the same result in different ways, or making assumptions to produce a solution may not have been given much attention. As in the examination made according to the principles of MEA, the model documentation principle emerges as a justification that is not emphasized much by the pre-service teachers.

It is noteworthy that among the justifications provided by pre-service teachers for why the example is an MEA, emphasis is scarcely placed on the modelling process being cyclic. The fact that most of the activities that were developed in fact do require a cyclic process may have caused the participants to perceive this as a natural process and prevented them from emphasizing this



property. Most of the MEA, developed or chosen from the literature, remained within the field of mathematics. Consequently, the use of interdisciplinary knowledge was also not a common justification.

It is observed that there is high consistency between the justifications given by the participants and the problems they wrote. This indicates that the properties of the modelling activities are understood correctly by the participants. A significant difference is seen only regarding the real-life context. There could be two reasons for this. First, examples not completely compatible with the real-life context were not included in the frequency. Another explanation relates the pre-service teachers' confusion in evaluating the real-life context to the word problems they are accustomed to from textbooks.

It is thought that the results of this study will provide insight to the faculty members regarding the structuring of the practice-oriented part of the modelling course in the new undergraduate mathematics curriculum. The perceptions of pre-service teachers towards the real-life context in word problems emerged as one of the issues that need to be considered in modelling education. Although modelling activities being suitable for creating a model at the end has a high frequency, the model documentation principle, which reveals the thinking processes of the students, is not given enough importance in attaining this model. This points to the need to raising attention regarding this issue. It may be necessary to draw the attention of pre-service teachers to the importance of this issue and its benefits to students. Further, faculty members may be careful to choose activities in accordance with this principle for the MEA they will use during modelling education in class.

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